Performance Prediction Model for a High-Impulse Monopropellant Propulsion System

H. C. HEARN* AND D. L. YOUNG†
Lockheed Missiles & Space Company, Inc., Sunnyvale, Calif.

A performance model has been developed for use with a high-impulse, multistart monopropellant propulsion system. In developing the mathematical model, theoretical considerations plus empirical data correlations were employed to characterize the propellant tank blowdown, feed system operation, and thruster performance. The system model derivation involved the development of a set of differential equations which can be numerically integrated to predict propellant consumption, pressure decay, and thruster performance for selected mission profiles. The significant feature of the model is its ability to characterize catalyst bed changes with thruster life, enabling performance prediction for a hydrazine system with extensive capabilities and a 3 : 1 blowdown pressure range. An accuracy of better than 2.0% has been demonstrated in system testing.

Nomenclature

A = area
B_1, B_2, B_3, B_4 = nitrogen solubility coefficients
C^* = characteristic exhaust velocity
C_D = Venturi discharge coefficient
E = Young's modulus of elasticity
F = thrust
g_o = standard gravitational acceleration
h = tank wall thickness
I = impulse
I_{sp} = specific impulse
K = flow coefficient
N_1, N_2, N_3 = hydrazine vapor pressure coefficients
n = polytropic exponent
P = pressure
r = tank radius
S = nitrogen solubility
T = temperature
t = time
V = volume
V_o = tank volume at standard conditions
W = mass
$\dot{w}$ = propellant flowrate
α = coefficient of thermal expansion
μ = Poisson's ratio
ρ = propellant density
σ = standard deviation

Subscripts

B = catalyst bed
C = thrust chamber
H = hydrazine vapor
i = initial conditions
L = liquid
N = nitrogen
P = propellant
T = tank

Introduction

EXPERIENCE with monopropellant hydrazine thrusters has thus far been limited to low thrust attitude control and minor trajectory corrections involving modest impulse requirements. A high impulse system incorporating an engine with 250 lb (1112 N) maximum thrust with multistart capability has now been developed; this necessitated the synthesis of a performance model which could be used to predict the engine firing time required to produce a desired impulse. The model would have to demonstrate a high degree of accuracy for a thruster capable of at least 45 ambient temperature starts while operating over a variable duty cycle involving a total impulse greater than 650,000 lb-sec (2,890,000 N-sec).

It is known that the catalyst beds employed in hydrazine thrusters experience physical changes involving catalyst activity, average particle size, particle migration, and void formation as life accumulates on the thruster. This aspect requires a procedure which can be updated to reflect the effects of these changes on performance. For this particular application, the situation is also complicated by a decaying propellant feed pressure; therefore, the changing life effects are superimposed on a continually decreasing thrust level. These considerations were incorporated into an operational model and digital computer program having both impulse and propellant mass status predictability.

System and Component Description

The system is shown schematically in Fig. 1 and features a 3 : 1 blowdown pressurization system. The propellant feed system

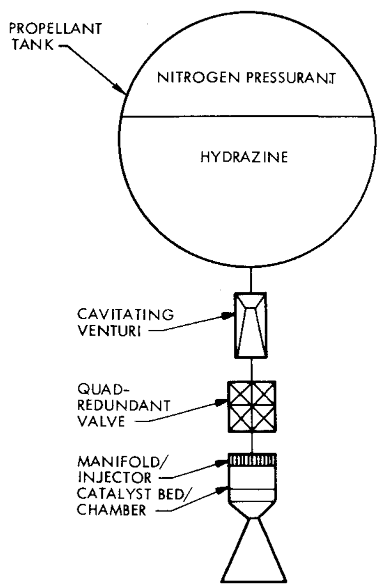

Fig. 1 Monopropellant propulsion system schematic.

Received March 25, 1974; revision received June 14, 1974. The authors wish to acknowledge the contribution of C. S. Marantz to the original mathematical model.

Index categories: Liquid Rocket Engines; Spacecraft Propulsion Systems Integration.

* Research Engineer, Propulsion Systems, Space Systems Division. Member AIAA.

† Programer-Analyst, presently with System Development Corp., Colorado Springs, Colorado.

includes a cavitating venturi which decouples chamber pressure oscillations from feed line pressure surges and improves system impulse predictability. The thruster itself produces from 250–140 lb (1112–623 N) thrust depending on feed pressure, and uses Shell 405 catalyst to promote hydrazine decomposition.

Pressurization

The system is pressurized by nitrogen which blows down from 300 psia to 100 psia. Hydrazine vapor is also present in the propellant tank and its contribution to tank pressure is accounted for in the model. Other factors included in modeling the pressurization system are heat transfer from the surroundings to the expanding gas, nitrogen solubility in hydrazine, and tank volume changes due to temperature and pressure effects.

Tank pressure is the sum of nitrogen partial pressure and hydrazine vapor pressure. For generality the Beattie-Bridgeman equation of state for nitrogen is used. Although introducing some complexity in a pressure and temperature range where the perfect gas equation of state would be acceptably accurate, the Beattie-Bridgeman equation permits extension of the model without change to temperature and pressure ranges beyond the scope of the perfect gas law. To account for cooling due to expansion of the nitrogen a polytropic process is assumed. Nitrogen pressure and ullage volume are related by

$$P_N V_N{}^n = P_{N_i} V_{N_i}{}^n \tag{1}$$

The value of the polytropic exponent was chosen based on heat-transfer considerations and experience with other systems. Close agreement between test data and model predictions indicates satisfactory modeling of the heat transfer process. Hydrazine vapor pressure is expressed as an exponential function of propellant temperature, and is based on physical property data.[1] The form of the equation is

$$P_H = \exp\left(N_1 + \frac{N_2}{N_3 + T_P}\right) \tag{2}$$

Hydrazine vapor pressure is assumed to remain constant during an engine firing since the large heat capacity of the propellant should preclude temperature changes.

Solubility of nitrogen in hydrazine is also an exponential function of hydrazine temperature, but multiplied by nitrogen pressure. The relationship

$$S = B_1 P_N \exp\left(B_2 + B_3 T_P + B_4 T_P{}^2\right) \tag{3}$$

is employed in the model. Tank volume changes due to temperature and pressure are modeled by assuming the tank to be a thin-wall spherical pressure vessel of uniform wall thickness. Using first-order linear elasticity corrections the tank volume is given by

$$V_T = V_o[1 + 3/2(Pr/Eh)(1-\mu) + 3\alpha\Delta T] \tag{4}$$

Feed System

The propellant feed system contains a cavitating venturi. Although the primary purpose of the venturi is to decouple chamber pressure oscillations from feed line pressure surges, an additional advantage is that modeling of system performance is made easier and more accurate. The cavitating venturi renders propellant flowrate dependent only on propellant vapor pressure and feed pressure at the venturi inlet. The flowrate under cavitating conditions can be expressed simply as

$$\dot{w} = C_D A[2g_o \rho(P_T - P_H)]^{1/2} \tag{5}$$

Test data showed that the discharge coefficient of the venturi can be modeled as a second order polynomial function of the Reynolds number (and hence flowrate) based on throat diameter. The equation for flowrate then becomes a complicated function of viscosity, venturi throat diameter, propellant density, propellant vapor pressure, and tank pressure. The solution, however, can still be expressed in closed form.

Normal pressure decay during blowdown operation in conjunction with increased pressure drop across the catalyst bed can preclude flow cavitation at the venturi. Although not a feature of nominal operation, decavitated flow has occurred during testing, and is included in the model. A pressure balance is used to relate tank pressure, chamber pressure, and the pressure drops across the catalyst bed and the other flow passages in the system.

$$P_T = \Delta P_L + \Delta P_B + P_C = K_L \dot{w}^2 + K_B \dot{w}^{3/4} + \frac{C^* \dot{w}}{A g_o} \tag{6}$$

Equations (5) and (6) are used jointly to determine whether cavitated or decavitated flow exists. For a given tank pressure Eq. (5) is used to determine a flowrate for cavitating flow. Equation (6) is then used to determine whether the cavitated flow is consistent with the given tank pressure. If decavitated flow is indicated, an iterative technique is employed to solve Eq. (6) for the proper value of flowrate to be used in the model.

Thruster Performance

Modeling of the thruster was first attempted on a theoretical basis, but proved to be infeasible due to the complexity of relationships between thruster performance and operating conditions such as catalyst bed temperature, pressure drop across the bed, hydrazine residence time in the bed, and the amount of ammonia dissociation. Modeling is further complicated by the fact that the system never operates at steady-state conditions; there is a start transient followed by a period of slow thrust decay due to decreasing tank pressure.

Modeling of the thruster therefore was done empirically by performing regression analyses on data involving various performance parameters. The parameters modeled include specific impulse (I_{sp}), characteristic exhaust velocity (C^*), and catalyst bed resistance factor (K_B), defined as pressure drop across the catalyst bed divided by propellant flowrate raised to the three-fourths power, ($\Delta P_B/\dot{w}^{3/4}$). Thruster performance is sensitive to life effects (accumulated impulse and duty cycle) so it is necessary to find a parameter which reflects these effects. The bed resistance factor proved to be a sensitive and convenient parameter for describing catalyst bed changes. Figure 2 shows the thruster schematic and the location of the pressure transducers used to determine catalyst bed pressure drop. The bed resistance factor undergoes change as impulse is accumulated; the rate of change depends strongly on the duty cycle experienced by the thruster plus the propellant flow distribution in the catalyst bed.

The bed resistance factor at twenty sec into a burn is chosen as the parameter which characterizes cumulative life effects. This value (K_{20}) is used as an input parameter to shape the model for bed resistance factor (K_B) which describes the catalyst bed state during a given burn. The model which results from regression analyses takes the form

$$K_B = f(\dot{w}, t, K_{20})$$

The abovementioned empirical model for K_B is itself used in the models for C^* and I_{sp}.

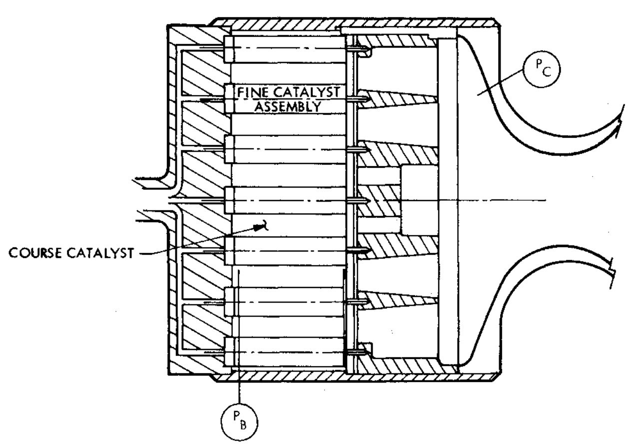

Fig. 2 Hydrazine thruster.

$$C^* = f(\dot{w}, t, K_B)$$
$$I_{sp} = f(\dot{w}, t, K_B)$$

A single set of equations cannot adequately model both the start transient and the nearly steady-state portions of a burn. Transient performance equations model the first 35 sec of a burn, after which a different set of "steady-state" equations is employed.

All of the independent variables in the regression analysis reflect the degree of ammonia dissociation in the exhaust gas. When hydrazine decomposes the products are ammonia, nitrogen, and heat. Some of the ammonia then dissociates into nitrogen and hydrogen and absorbs heat released during decomposition. Thruster performance is degraded as more ammonia dissociates, and engine design generally attempts to minimize ammonia dissociation. The extent of dissociation is determined by the temperature of the thrust chamber components (principally the catalyst bed) and by the residence time of decomposition products in the catalyst bed. Temperature of the thrust chamber depends on the elapsed burn time and the hydrazine throughput (hence flowrate). Residence time of hydrazine in the catalyst bed depends on flowrate and bed impedance, which are reflected in the bed resistance factor. Finally, the use of K_{20} establishes a fixed point condition through which the general bed resistance relationship must pass.

Performance Model Derivation

The elements previously described are combined to create a complete system model which is used to predict thruster burn time and other parameters describing system performance and status. The final result is a set of differential equations which can be solved for the desired information.

The first equation is obtained by writing the differential of the nitrogen partial pressure

$$dP_N = \frac{\partial P_N}{\partial T_N} dT_n + \frac{\partial P_N}{\partial W_N} dW_n + \frac{\partial P_N}{\partial V_N} dV_N \tag{7}$$

In principle, some nitrogen will come out of solution during the blowdown process, since the amount of nitrogen dissolved in the hydrazine is proportional to the tank pressure. However, investigations conducted during the development program revealed that the rate at which gas comes out of solution is very low for small changes in pressure. In view of these results, and considering that the blowdown time will be a few seconds or minutes, the change in pressurant mass during the blowdown is assumed to be negligible. Holding this nitrogen mass constant for a given burn reduces the number of differential equations required and simplifies the final solution. The first equation in the solution set then becomes

$$dP_N - \frac{\partial P_N}{\partial T_N} dT_N - \frac{\partial P_N}{\partial V_N} dV_N = 0 \tag{8}$$

Although the nitrogen mass during a burn is held constant, the model does account for changes during a mission. When the performance model computer program is used for mission simulation purposes, the mass is adjusted before each burn in order to account for the current nitrogen solubility and the amount of dissolved nitrogen expelled during the previous burn.

The tank volume is expressed as the sum of propellant volume and ullage volume

$$V_T = V_N + V_P = V_N + W_P/\rho \tag{9}$$

The differential of tank volume is a function of pressure and temperature changes

$$dV_T = \frac{\partial V_T}{\partial P_T} dP_T + \frac{\partial V_T}{\partial T_T} dT_T = dV_N + \frac{1}{\rho} dW_P - \frac{W_P}{\rho^2} d\rho \tag{10}$$

As noted previously, heat transfer during a blowdown is insufficient to cause significant changes in the temperature of the propellant mass or the tank wall. Therefore, the propellant temperature during a burn is assumed to be the initial propellant temperature (equal to tank wall temperature). The propellant density is a function of both temperature and tank pressure. Since pressure changes during a burn are not large, and propellant temperature is held constant, the hydrazine density during a burn is held constant. The tank pressure can be written as the sum of nitrogen partial pressure and hydrazine vapor pressure; therefore

$$dP_T = dP_N + dP_H \tag{11}$$

A volatile liquid pressurization study[2] has shown that the hydrazine liquid and vapor can be assumed to remain in equilibrium for the time periods under consideration. Since hydrazine vapor pressure is a function only of propellant temperature (assumed to be constant), the partial pressure of hydrazine vapor remains constant during a burn. Using the assumptions described above, and combining Eqs. (10) and (11), the second differential equation for the system is obtained

$$\rho(\partial V_T/\partial P_T)\, dP_N - \rho\, dV_N = dW_P \tag{12}$$

The third system equation is obtained by differentiating and rearranging the polytropic blowdown relationship [Eq. (1)].

$$n \frac{V_{N_i}}{P_{N_i}} \left(\frac{P_{N_i}}{P_N}\right)^{1/n+1} dP_N + dV_N = 0 \tag{13}$$

Introducing the relationship

$$F = \frac{dI}{dt} = -\frac{dW_P}{dt} I_{sp} = \dot{w} I_{sp},$$

and solving the previous Eqs. 8, 12, and 13 simultaneously, the complete set of performance model differential equations becomes

$$\frac{dP_N}{dI} = -\frac{1}{I_{sp}(C_3 + C_4 C_5)} \tag{14}$$

$$\frac{dT_N}{dI} = -\frac{1 + C_2 C_5}{I_{sp} C_1 (C_3 + C_4 C_5)} \tag{15}$$

$$\frac{dV_N}{dI} = \frac{C_5}{I_{sp}(C_3 + C_4 C_5)} \tag{16}$$

$$\frac{dt}{dI} = \frac{1}{\dot{w} I_{sp}} \tag{17}$$

$$\frac{dW_P}{dI} = -\frac{1}{I_{sp}} \tag{18}$$

where

$$C_1 = \partial P_N/\partial T_N$$
$$C_2 = \partial P_N/\partial V_N$$
$$C_3 = \rho(\partial V_T/\partial P_T)$$
$$C_4 = \rho$$
$$C_5 = \frac{V_{N_i}}{n P_{N_i}} \left(\frac{P_{N_i}}{P_N}\right)^{1/n+1}$$

With differential equations for P_N, V_N, T_N, W_P, and t in terms of I, instantaneous values of all parameters of interest can be calculated by a numerical integration procedure. A fourth-order Runge-Kutta technique is employed to solve the equations for the first four impulse increments, whereupon Hamming's predictor-corrector method is used until the desired total impulse is attained.[3,4] The abovementioned partial derivatives and other values required for the differential equations are determined from the relations obtained for the tank and pressurization system. Thruster I_{sp} is determined from the empirical model previously described. At the conclusion of the calculation sequence, values of all parameters have been generated for each impulse increment in the burn, and the total burn time has been established. The mass status of the tank is also available, since this is continually determined by flowrate integration.

Performance Model Implementation

The performance model computer program is designed for use in three different applications: a) flight simulations; b) post-

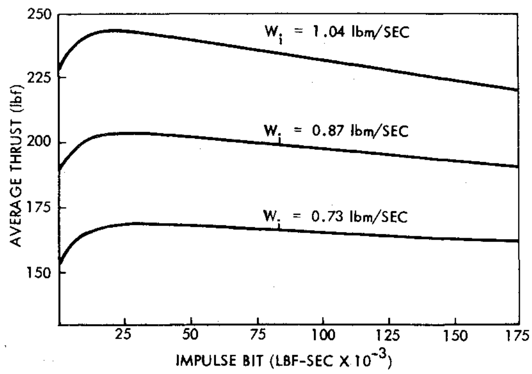

Fig. 3 Influence of impulse bit on average thrust.

firing evaluation and performance model improvement; and c) operational burn time prediction. In the simulation mode, the program is used to investigate propellant consumption, pressure decay, and predicted thruster performance for selected mission profiles. This provides a useful tool for determining whether mission objectives can be met with nominal thruster operation when the required total impulse and/or other aspects of the system are altered. The inputs for the simulation are the loading conditions, assumed K_{20} history based upon test data, propellant temperature, and the desired impulse requirements for each burn. The program calculates and tracks the various system parameters and determines the initial conditions for each burn based upon the conditions at the end of the preceding burn. For postfiring analysis and model evaluation, the actual pressures, temperatures, and values of K_{20} determined by instrumentation are used as inputs along with the actual burn times for each engine start. The impulse and mass status predictions are compared to the measured values to check the accuracy of the model as well as to update and improve the performance model routines.

The most important use for the model is in operational burn time prediction where actual tank pressures and temperatures, the estimated value of K_{20} for the burn, and the impulse required are inputs to the prediction procedure. The output is the predicted burn time required for the thruster to deliver the desired impulse. Data analysis during thruster development showed that the total impulse produced when the catalyst bed is initially hot is slightly greater than the impulse produced for an ambient temperature engine start. An empirical equation was developed to adjust the predicted burn time for various catalyst bed temperatures. This burn time correction is important for hot starts with impulses up to approximately 15,000 lb-sec (66,700 N-sec) but becomes negligible for greater impulse bits.

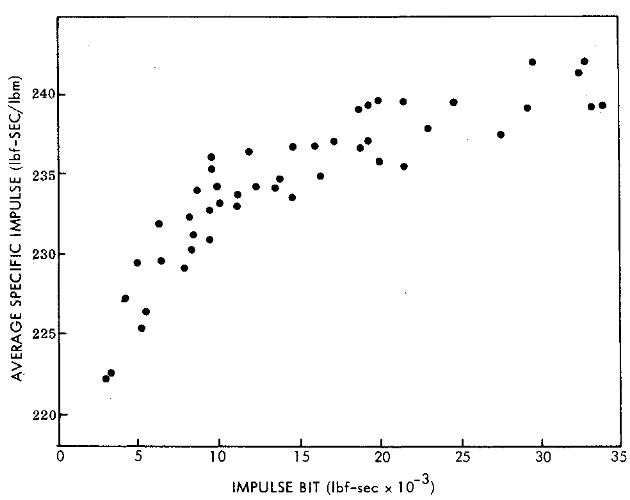

Fig. 4 Influence of impulse bit on specific impulse.

In addition to the previous application, the performance model was used to develop a simplified performance prediction method; this method involves empirical equations derived from computer simulations and test data analysis. In this procedure the average thrust and specific impulse for a burn are defined as

$$\bar{F} = \int F\,dt/\Delta t = I/\Delta t$$

$$\bar{I}_{sp} = \int F\,dt/W_p = I/W_p$$

where Δt refers to the total burn time and W_p the propellant consumption. These simplified empirical models were created so that burn time and propellant consumption predictions could be performed using only initial temperature and pressure data plus the desired impulse prior to a burn. The variables employed in the equations are impulse, predicted initial flowrate, and the estimated value of K_{20} for the burn.

A multiple regression analysis was performed on data generated by the performance model computer program to determine the significant functional relationships. The relationship of average thrust to flowrate and impulse bit is shown in Fig. 3, which is based on results from the performance model. The relationship between average specific impulse and impulse bit is shown in Fig. 4, which represents data from actual firings. The figures also give an indication of the transient nature of the thruster performance. For operational purposes, the empirical procedure offers the advantage of efficient performance prediction with accuracy only slightly less than that of the full system model computer program. If major changes are required in the propellant load or pressurant mass, new equation coefficients can be rapidly generated through use of the performance model computer program and a regression analysis program.

Performance Prediction Accuracy

A comprehensive error analysis was conducted to determine the impulse predictability expected under operational conditions. Uncertainties were determined for the tank blowdown, feed system, and thruster performance models, then combined with the instrumentation uncertainties to obtain the prediction accuracy. This study showed that the 2σ impulse predictability error should be within 2% for burns involving total impulses greater than 10,000 lb-sec (44,500 N-sec). The percentage error increases for shorter burns and is a function of the delivered impulse.

Table 1 summarizes the performance characteristics and the extensive experience which has been obtained thus far on the propulsion system. Approximately 75% of the firings have been single ambient starts with the catalyst bed temperature in the 120°F–140°F (322 K–333 K) range. The other burns were pair firings which consisted of an ambient start followed by a catalyst bed cooldown to 300°F–800°F (422 K–700 K) and then another engine start. The highest temperatures were associated

Table 1 Performance summary

Parameter	Value	
Number of system tests	6	
Number of thruster firings	184	
Accumulated total impulse	2,722,000 lb_f-sec	(12,108,100 N-sec)
Total burn time	14,702 sec	
Total propellant consumption	11,554 lbm	(5241 Kg)
Average specific impulse	235.6 lb_f-sec/lbm	(2310.6 N-sec/Kg)
Impulse bit range	800–165,000 lb_f-sec	(3560–734,000 N-sec)
Average thrust range	145–243 lb_f	(645–1080 N)
Initial catalyst bed temperature	122–1466°F	(323–1070 K)
Impulse predictability (2σ)		
$I >$ 10,000 lb_f-sec	1.82%	
$I <$ 10,000 lb_f-sec	2.36%	

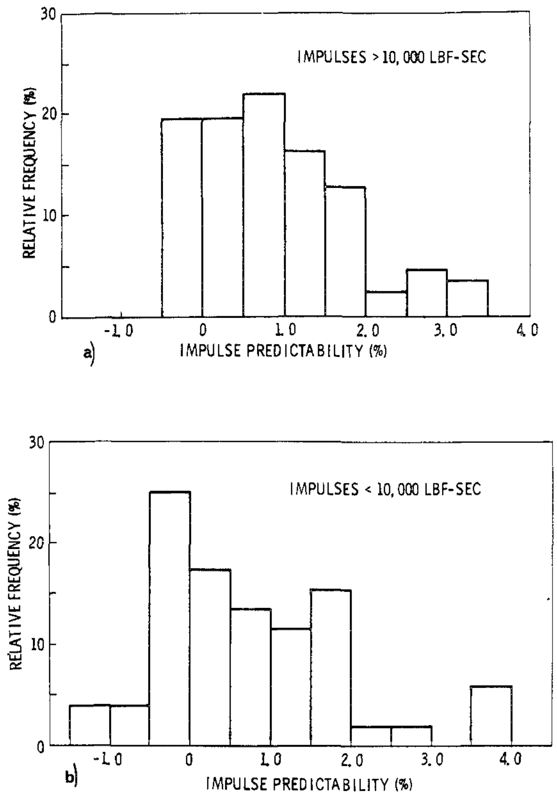

Fig. 5 Impulse predictability from actual firings.

with tests of the thruster in the pulse mode, with off times of only five sec.

The test data obtained to date has demonstrated the usefulness of the catalyst bed pressure drop and bed resistance factor in characterizing thruster life effects for the purpose of predicting performance. The data is also sufficient to validate the propulsion system performance model and prediction procedures over a wide range of operating conditions. Figure 5 presents the results of an impulse predictability analysis conducted using the data. It is apparent that a bias of approximately one percent appears in the predictions. This bias, due to some recurring error source in the model, is being evaluated, but for the present is satisfactorily accounted for by including the factor in all predictions. As shown in Table 1, the 2σ error is only 1.82% for burns involving impulses greater than 10,000 lb-sec. The predictions are in very close agreement with the delivered impulses, indicating that the system model can be employed confidently for operational performance predictions.

Conclusions

A performance model which yields accurate burn time predictions has been developed and validated for use with a high-impulse, multistart monopropellant propulsion system. The significant feature of the model is its ability to characterize catalyst bed changes with thruster life and incorporate these effects into the performance prediction. System testing has demonstrated the accuracy of the techniques involved.

References

[1] Deeg, W. E., "Establishment of Physical Characteristics of Anhydrous Hydrazine," LMSC Rept. 72898, June 1967, Lockheed Missiles and Space Co. Inc., Sunnyvale, Calif.

[2] "Final Report, Volatile Liquid Pressurization System Independent Development Program," LMSC Rept. A965247, Jan. 30, 1970, Lockheed Missiles and Space Co. Inc., Sunnyvale, Calif.

[3] Henrici, P., *Elements of Numerical Analysis*, Wiley, New York, 1964, pp. 263–288.

[4] Southworth, R. W. and Deleeuw, S. L., *Digital Computation and Numerical Methods*, McGraw-Hill, New York, 1965, pp. 453–460.